Applied Medical Informatics
Case study Vol. 31, No. 4/2012, pp: 42-50

# Application of the Warping Methods in Dental Radiology Images Analysis

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**Received: 23 September 2012/Accepted: 10 November 2012/ Published online: 24 November 2012**

**Abstract**
The "image warping" functions are useful techniques to match image objects when there are strong correlations among the positions of these objects in space. In order to do this, a function *f*, called the warping function, is described in specialized literature and is used to convert a given image into a reference known image. Several types of warping functions are described in the specialized literature – affine function, Procrustes transformation, bilinear transformation, perspective transformation and mosaic transformation. In this paper, we present these functions and apply them to process several dental radiology images in a controlled manner, defined by a variation range of parameters.

**Keywords:** Warping function; Dental radiology; Image processing; Median filter; Adaptive filter.

## Introduction

The warping of images represents an important stage in many applications of image analysis. This method of processing aims at transforming two-dimensional images geometrically in order to distort the object shapes. The distortion degree can vary from minimal modifications up to radical changes, and the method of distortion, at its turn, can vary from the simplest changes – such as scaling or rotation, to irregular or combined warping.

From a technical point of view, warping is done through the transformation of all pixels in a given image using a two-dimensional function applied to them; such a function will change the pixels position on the image and will usually leave the colors unmodified. The image warping is more powerful when the applied function is more complex in structure.

This technique of image processing has many applications. Practically, even the simplest software packages for digital images processing require at least one transformation of the image through scaling, rotation or moving – all of these being basic examples of image warping.

One of the oldest but very important applications of image warping consists in its identification by referring it to a reference pattern, like a map [1, 2]. There are algorithms to correct satellite images – because the photographs taken this way are usually affected by some geometrical distortions caused by the lens imperfections, perspective or curvature of the earth; these distortions can be mathematically expressed through certain functions, and therefore they can be eliminated by applying the inverse functions to the image.

 

Another application of this technique is known as "texture mapping" [3] and it is widely used in 3D computerized graphics: in order to obtain a more natural aspect of the 3D automatically generated objects, we can apply on their surface some bi-dimensional textures which simulate certain features: wood grain, marble ribs, skin, and surface bumpiness.

A derived and more spectacular application is the one known as "morphing" (from metamorphosis) [4]. The image warping functions are combined with key-frame animation and cross-fading in order to create a convincing illusion of a smooth transformation of an object into another.

A generalization of image warping is also widely used: the warping of one-dimensional signals in order to obtain their alignment / synchronization. Known as the "dynamic time warping", this technique has applications in speech processing, handwriting analysis, alignment of tracks in electrophoresis gels and so on.

Another application of image warping concerns the medical field. The general concept is to identify / recognize the recorded images using different devices for medical investigation, by referring them to standard images taken from medical atlases [5].

In order to establish the diagnosis, the physician needs to work with very clear images having good contrast, brightness and quality, so he may identify the so-called "areas of radio-transparency" which show modifications in the bone structure, usually determined by an infectious process. Another point of interest is to identify as accurately as possible the shape and limits of these areas, which show the nature of the infectious process – closed (closed granuloma) or extended (diffuse areas). Our subject of interest is included in the category of medical images processing, the aim of our study being to enhance the quality of dental radiology images by using certain warping functions in a clearly-controlled manner.

## Material and Method

The warping is defined as a mapping from the coordinate space of a source image (x, y) to the coordinate space of a destination image (u, v), using a pair of two-dimensional functions f, g: f(x, y) = u, g(x, y) = v (where x denotes the column number and y denotes the row number).

As a matter of fact, this sort of mapping is called "forward mapping" [6], which is a many-to-one mapping, used to model the distortions where several points in the source image map to the same point in the destination image. This warping is usually performed by scanning the source image pixel by pixel, calculating the corresponding location in the destination image by evaluating the mapping function, and painting that location in the destination image with the color of the source pixel.

The transformations made through image warping are classified in two main categories:

- Parametric transformations
- Non-parametric transformations.

The parametric transformations can be classified, according to their complexity, in the following categories of transformations [7]:

1. Translation:

$$\begin{aligned} u &= x + a \\ v &= y + b \end{aligned} \qquad (1)$$

(this change being applied only along the rows, only along the columns or along the rows and columns simultaneously). The top-left corner (0, 0) of the source image matches with the location (a, b) of the destination image.

2. Translation and dilation (scale change with a fixed factor c):

$$\begin{aligned} u &= cx + a \\ v &= cy + b \end{aligned} \qquad (2)$$

(the value $c = 1$ corresponds to no change in magnification, the values $c > 1$ represent enlargements, while the values $c < 1$ represents shrinkages).

3. Translation and rotation (angle change with a fixed value θ):

$$u = x \cdot \cos\theta + y \cdot \sin\theta + a \qquad (3)$$

$v = - x \cdot \sin \theta + y \cdot \cos\theta + b$

4. Euclidean / Procrustes transformation (magnification change and a rotation of θ degrees):

$$\begin{aligned} u &= cx \cdot \cos\theta + cy \cdot \sin\theta + a \\ v &= - cx \cdot \sin \theta + cy \cdot \cos\theta + b \end{aligned} \qquad (4)$$

This is a four-parameters transformation, which can be uniquely defined from two points in the two images.

5. Affine transformation (1st order polynomial):

$$\begin{aligned} u &= a_1 \cdot x + a_2 \cdot y + a \\ v &= b_1 \cdot x + b_2 \cdot y + b \end{aligned} \qquad (5)$$

This is a six-parameters generalization of the Procrustes transformation, which allows different stretching along rows and columns of an image and its shearing. An orthogonal pair of directions in the x – y image remains orthogonal in the u – v image, and the transformation either stretches or shrinks in these two directions.

6. Perspective transformation:

$$\begin{aligned} u &= \frac{a_1 x + a_2 y + a}{c_1 x + c_2 y + 1} \\ v &= \frac{b_1 x + b_2 y + b}{c_1 x + c_2 y + 1} \end{aligned} \qquad (6)$$

This type of transformation appears in order to show the situation when a planar object is viewed from a fixed point in space. It is a nonlinear transformation requiring eight parameters, being a generalization of the affine transformation. It becomes affine when the viewing point is situated to a large distance – in this case the foreshortening effects tending to zero. The perspective transformation is the most general transformation which maps straight lines at all orientations to straight lines and preserves conic sections (circles, ellipses, parabolas and hiperbolas). As in all the previous transformations, the perspective is functionally invertible and also bijective.

7. Bilinear transformation:

$$\begin{aligned} u &= a_1 \cdot x + a_2 \cdot y + a_3 \cdot xy + a \\ v &= b_1 \cdot x + b_2 \cdot y + b_3 \cdot xy + b \end{aligned} \qquad (7)$$

This is also an eight-parameters transformation which generalizes the affine transformation, being instead different by the perspective transformation. The straight lines in three particular directions are preserved, including lines parallel to the x and y axis. This transformation is not rotationally invariant – when both images are rotated, the transformation between them would be different – and it is not necessarily bijective.

8. Polynomial and other transformations:

$$\begin{aligned} &u = \sum_{i=0}^{p}\sum_{j=0}^{p-i} a_{ij} x^i y^j \text{ , } v = \sum_{i=0}^{p}\sum_{j=0}^{p-i} b_{ij} x^i y^j \text{ or} \\ &u = \sum_{i=0}^{p}\sum_{j=0}^{p} a_{ij} x^i y^j \text{ , } v = \sum_{i=0}^{p}\sum_{j=0}^{p} b_{ij} x^i y^j \end{aligned} \qquad (8)$$

These transformations include quadratic, biquadratic, cubic and bicubic functions as special cases. There are also many alternative parametric transformations to polynomials: the mapping of arcs of concentric circles to straight lines in order to remove the flexion from images of fishes; matching landmarks for spiral structures and other patterns of growth; aligning of hand-drawn templates with objects in images; removing of warping from multi-track electrophoretic gels by estimating the orientation direction of bands in different parts of a gel with polynomial functions.

9. Mosaicing transformation:

In image mosaicing, the transformation of images is not necessarily known beforehand. In the example given in the next section, two images are merged and we will estimate the transformation by letting the user give points of correspondence (also called landmarks or fiducial markers) in each of the images. In order to recover the transformation we rearrange the warping equation $\vec{x}' = T\vec{x}$ so that the warping parameters is the vector $\vec{t}$ in $\vec{x}' = Z\vec{t}$ .

In matrix form the above system becomes:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x \; y \; 100 \\ y - x010 \\ 0 \; 0 \; 001 \end{bmatrix} \cdot \begin{bmatrix} s \cdot \cos\alpha \\ s \cdot \sin\alpha \\ t_x \\ t_y \\ 1 \end{bmatrix} \tag{9}$$

The warping parameters are obtained by solving the linear system given above. Precisely, we have the solution $\vec{t} = Z^{-1} \cdot \vec{x}'$. We notice that (at least) two points are required to determine the four warping parameters.

The non-parametric transformations ([8]) are a more complex category of warping functions, designed mainly to solve the drawbacks of parametric transformations – for example, the problem of local distortions on an image. There are many types of non-parametric transformations, the most frequently used being: the first-order or linear spline (used to achieve the Delaunay triangulation between a set of matched landmarks in two images), the cubic spline (used to produce a smoother transformation), the elastic deformations and the thin-plate splines.

1. The elastic deformations:

In order to introduce smoothness constraints, the warping is made using the model of distortion of an elastic sheet or membrane. The elastic energy of a deformation is given by:

$$\iint \frac{1}{2}(w_{xx}\sigma_{xx} + w_{yy}\sigma_{yy} + 2w_{xy}\sigma_{xy})dxdy \tag{10}$$

where ($w_{xx}$, $w_{yy}$, $w_{xy}$) is the strain tensor,

$$w_{xx} = \frac{\partial u}{\partial x},\ w_{yy} = \frac{\partial v}{\partial y},\ w_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{11}$$

and ($\sigma_{xx}$, $\sigma_{yy}$, $\sigma_{xy}$) is the stress tensor,

$$\begin{aligned} \sigma_{xx} &= \frac{E}{1-\sigma^2}\left(\frac{\partial u}{\partial x} + \sigma\frac{\partial v}{\partial y}\right), \\ \sigma_{yy} &= \frac{E}{1-\sigma^2}\left(\frac{\partial v}{\partial y} + \sigma\frac{\partial u}{\partial x}\right), \\ \sigma_{xy} &= \frac{E}{2(1+\sigma)}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \end{aligned} \tag{12}$$

where E is Young's modulus and $\sigma$ is Poisson's ratio.

This model can be further particularized using different types of first-order differential equations. The approach can also be extended to three dimensions, for example to align three-dimensional medical images – as the X-ray computed tomography data with body atlases, or even to align a series of images in which there is a relative motion among several objects, using an analogy with fluid flow.

2. The thin-plate splines:

The warping transformation can be interpreted as a pair of two-dimensional surfaces, where u and v are represented as similar functions of x and y. These two surfaces are represented using a combination of radial functions and affine transformations [9]:

$$\begin{aligned} u &= \sum_{i=1}^{m} c_i f(\sqrt{(x-x_i)^2 + (y-y_i)^2}) + a_1 x + a_2 y + a, \\ v &= \sum_{i=1}^{m} c_i f(\sqrt{(x-x_i)^2 + (y-y_i)^2}) + b_1 x + b_2 y + b, \end{aligned} \tag{13}$$

where ($x_1$, $y_1$), ($x_2$, $y_2$), … ($x_m$, $y_m$) are a set of landmarks, and f is a well-defined function:

- Multiquadric: $f(t) = (t^2 + t_0^2)^\alpha$, $0 < \alpha < 1$;
- Shifted log: $f(t) = \log(\sqrt{t^2 + t_0^2})$, $t_0^2 > 1$;
- Gaussian density: $f(t) = \exp[-t^2 / (2\sigma^2)]$;
- Thin-plate spline: $f(t) = t^2 \log t^2$.

The choosing of *f* can be made in different ways, according to the warping's purpose. For example, f can be chosen to minimize some functional expressions – transformation used to align medical scan images, or the electrophoresis gels. When landmarks are distorted by different types of noise, the thin-plate splines can be used for smoothing. The method can be generalized by giving different weights to compressional and rotational transformations between images. Other authors also used the thin-plate spline to represent ground height, in combination with an affine transformation, to register an airborne synthetic aperture radar image with a digital map.

We used the image warping techniques in order to process dental radiology images. These images are recorded in grayscale and allow emphasizing and recognizing different elements which help to establish the final diagnosis. Our purpose was to distort the image in order to emphasize the areas of radio-transparency. All the processing was made in MATLAB, using the toolbox for Image Processing and writing our own procedures, based on the implemented functions.

**Results**

The first example initial image is shown in Figure 1. It is a classical orthopantomography for a female patient, aged 13, who presents upper diastema, upper lateral incisors hypodontia and an impacted upper right canine.

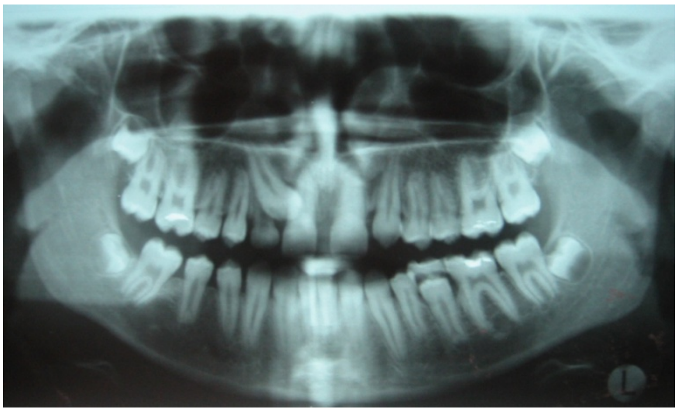

**Figure 1.** The initial image

The first step of our processing consisted in improving the contrast, the brightness and the saturation of the image, in order to better emphasize the grayscale and to obtain the the result shown in Figure 2.

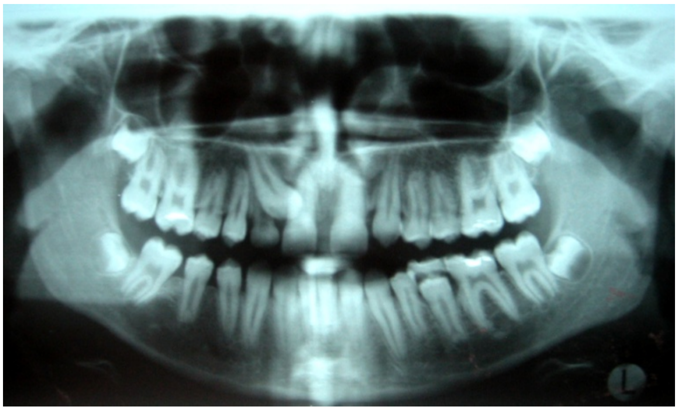

**Figure 2.** The image with contrast and brightness improved

Afterwards we sharpened the image, in order to emphasize the contours (the result is showed in Figure 3). The next step was to choose an area from the image (Figure 4) where the radio-transparency was present. There we applied an affine transformation after a spheric object, in order to better emphasize its characteristics (Figure 5).

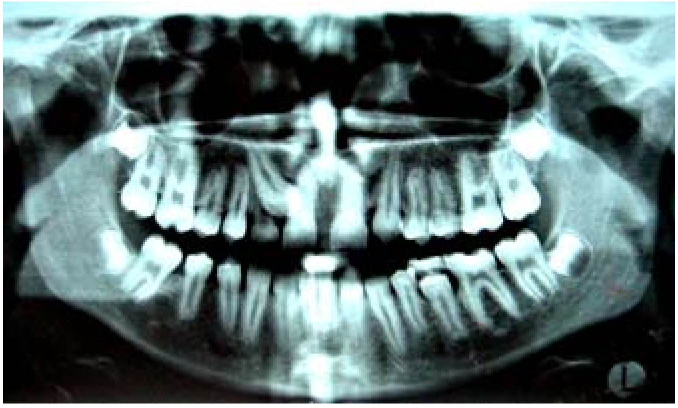

**Figure 3.** The sharpened image

Figure 4. **Selection of the interest area**

Figure 5. **The selected area after an affine transformation**

Likewise, a demonstration of mosaicing transform using two input images is given in Figure 6 a), b) and c) – the input images were selected as areas from Figure 1, too. The advantage of this technique of image processing consists in the possibility to reconstruct larger images from their component parts.

a) b) c)

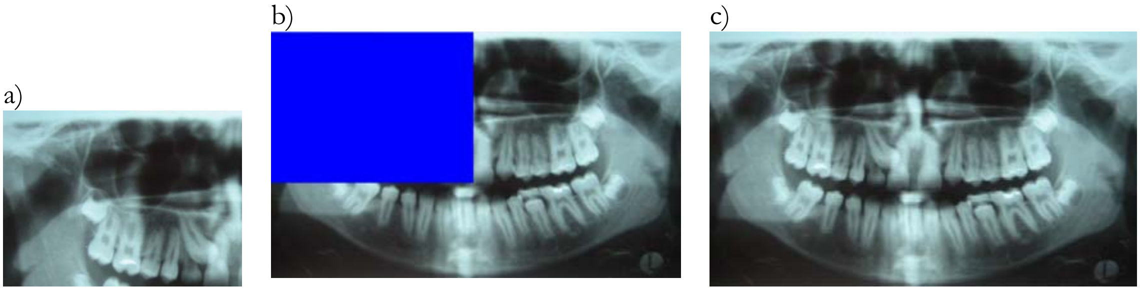

**Figure 6.** The mosaicking technique: a), b): component images; c) the resulted image

In the second example we processed a retro-dental radiography taken in order to check the correctness of the channel endodontic fillings. The initial image is presented in Figure 7.

Because the image was not very clear, and clarity was the most important feature we needed, we had to improve the contrast, brightness and saturation (Figure 8) and then we applied the sharpen effect (Figure 9), with better results – which allowed us to more accurately see the endpoints of the channel fillings in order to estimate their correctness from a medical point of view.

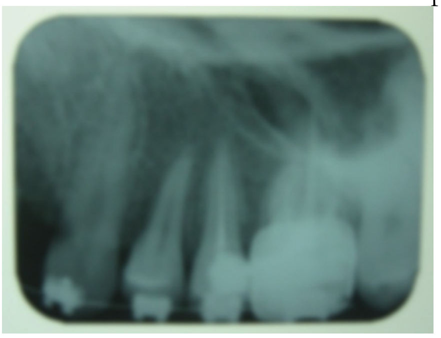

**Figure 7.** The initial image

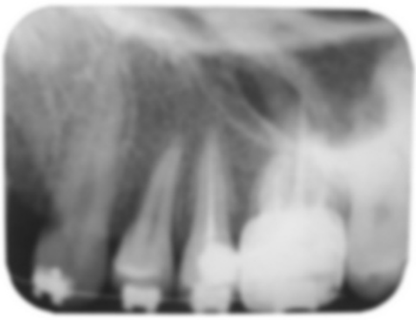

**Figure 8.** The contrast, brightness and saturation improved

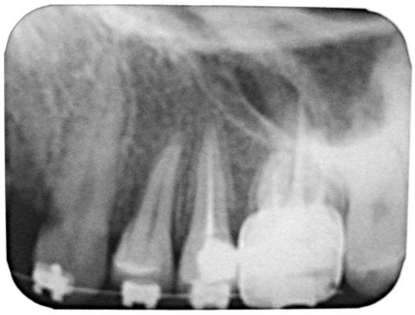

**Figure 9.** The sharpen effect

It also can be noticed that we preferred to desaturate the image (by keeping only the grayscale) because all the visual information we needed was covered by this range of colors – and, even more, the contours are more visible in grayscale.

A similar situation (regarding the image quality) is presented in the third example, where we had an occlusal dental radiography, made in order to depict the dental malpositions. The initial image is presented in Figure 10.

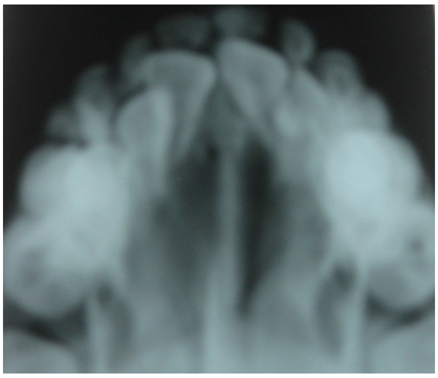

**Figure 10.** The initial image

This time the image was quite blurred, so the most important transformation we needed was its sharpening. First we tried to improve the contrast and the brightness (Figure 11), without desaturating the image, and then we applied a strong sharpen effect. The results can be seen in Figure 12.

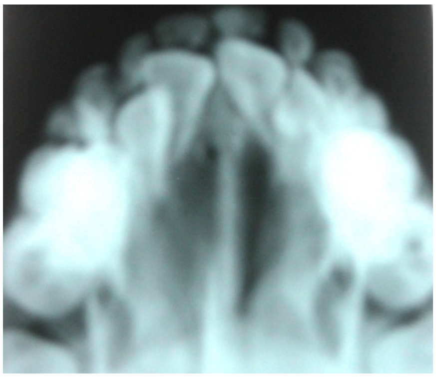

**Figure 11.** The contrast, brightness and saturation improved

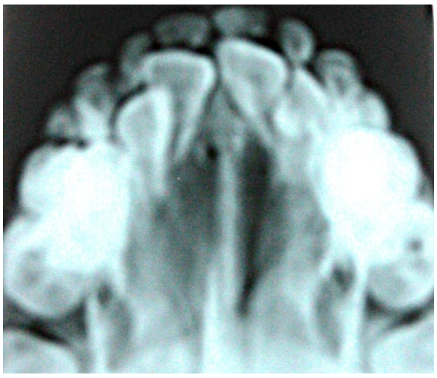

**Figure 12.** The sharpen effect

The image's quality was better, the tooth's position could be depicted more accurately, but we can notice that the image was still not perfect. In order to better define the image, more complex transformation algorithms may be required, in order to define better the contours and to eliminate the blur as much as possible.

## Discussion

A large number of applications concerning image warping techniques arise in many domains. While the drastic distortions are perfectly fitted in the area of visual effects (especially in cinematography and advertising, because of the spectacular and even shocking results of their use), tiny distortions of the captured images are sometimes necessary to be made in the high-tech scientific domains, like aerial photography via satellites, as well as medical sciences (electron microscopy, computerized tomography, images through magnetic nuclear resonance, X-rays), since they allow to emphasize certain specific features of the images and to correct some inherent errors generated by the nature of recording devices.

Image warping gives many possibilities to process the images emphasizing certain of their features. The main advantage of these methods is that, even if the mathematical definition of warping functions becomes sometimes quite complex, the practical results obtained using these functions provide accuracy, and allow to modify the image characteristics within a wide range of values, from the slightest changes - necessary only to emphasize certain aspects in the image - to the most radical ones, which drastically distort the image.

In our paper we presented a few examples about using the warping functions in order to improve the quality of dental radiology images – their contrast, brightness, saturation and sharpness, as standard features. We also tried to improve the image's sharpness using affine transformations applied to selected areas of interest – this method is helpful in order to emphasize the image's details and to allow the easier detection of radio-transparent areas. Finally, we used the mosaicing transformation in order to combine neighbor images in a single and larger one, a technique which is useful to emphasize the global aspect of an investigated area.

Image warping techniques became a forefront subject in the latest years, because they can produce spectacular effects in visual arts, mainly photography and film industry, as well as in the area of human face reconstruction or processing [6,10]. In the medical field, these techniques were applied especially in neurosurgery and in 3D reconstruction [5], and less in the dental radiology. There are several reasons for this. For example, this technique usually tends to produce radical changes in the initial image (the case of affine transformations); therefore, the risk to destroy significant details and to lose important information is increased, especially when the warping functions are applied at the level of the whole image. That is why it is important to work with this sort of transformations on previously selected areas of interest, which allow a better control of the details to be emphasized. These transformations are also quite complex; they require advanced knowledge of mathematics in order to define the appropriate parameters for warping functions, and a significant time to process each image, while the quantity of radiological dental images produced in a specialized unit, for example, is often too large in practice (a few dozen daily) to allow their appropriate processing with such tools.

In the examples presented in this paper we still used successfully warping functions to process dental radiological images, and it is obvious that the advantages of using these transformations cannot be denied. There are still some limits we did not overpasses yet, the most important being the missing of some well-defined quality standards in image processing (for example a minimal required level for contrast, brightness, saturation or sharpness) and the high degree of particularity for each processed image. Therefore, the images are processed using an empirical evaluation of their quality, and only the user's experience (technical as well as medical) can lead to good results.

A possible solution and our future goal is to automatized and to standardized this process, by integrating these warping functions in a specialized software, in order to obtain their improved and user-friendly control, to reduce as much as possible the degree of particularity and to establish some minimal quality standards necessary to evaluate the input images and the obtained results.

## Conclusions

The examples presented in this study showed clear improvements of the radiological dental images, which allowed to emphasize significant medical details, absolutely necessary in order to establish an accurate diagnosis. These types of image processing are also available in the professional software developed to work with images (like Corel PhotoPaint or Adobe Photoshop), but the advantage of making these processing by hand, using an exclusive mathematical approach, is an improved control of the output, because we can vary all the parameters of the transformation functions and their ranges of values in the most accurate way.

## Conflict of Interest

The author declares that he has no conflict of interest.

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