An Exploration of Mathematical Models for Tumor Growth

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Abstract

Introduction: Experimental research often involves measuring the effects of various interventions on tumor growth. Animal tumor models can be based on induced tumors or transplanted tumors. The growth patterns of such tumors are relevant in order to properly quantify the outcomes of various treatment interventions. Aim: The primary aim of this study was to explore systematically mathematical models for tumor growth and to characterize these models. Methods: Basic types included models based on empirical equations like the Michaelis-Menten equation, saturated exponentials, fractions of exponentials, as well as models based on differential equations, like logistic growth and polynomial fractions. Furthermore, various variations of the base models were compared to the base models. Linear combinations and nonlinear combinations of the primary models were also included in the analyses. The asymptotic volume of the tumor was considered fixed. Analysis was performed using the R statistical framework. Results: All models converged to the final tumor volume. However, variation of the model equations enabled the generation of a great diversity of growth curves with varying rates of convergence compared to the base types. The base models included the maximal volume and one additional rate parameter, while the variants included up to three modifying parameters. These models can be further combined using linear or nonlinear combinations to generate even greater diversity, with the drawback of doubling the number of parameters. A large log-likelihood can easily outgrow the increase in information criteria (like AIC or BIC) resulting from one or two more parameters in the simple variants. Conclusions: It was possible to construct many mathematical models for tumor growth starting from some basic types of non-linear or differential equations. Validation of these models should be performed on various sets of real data.

Keywords: Tumor Models; Logistic Growth; Saturating Exponential.

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